Formal centennial climate

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On December 12, 2015, 196 COP21 sovereign parties agreed to attempt to limit global warming to 1.5 degrees above preindustrial, without however specifying how to tell when that formal guard rail had been reached. We define a notion of formal centennial climate and propose it as a suitable such specification having the advantage of being predictable while agreeing with global HadCRUT5 as well as can be expected given the ocean's many subdecadal, decadal and multidecadal temperature oscillations. Formal centennial climate reaches 1.5 degrees above preindustrial in 2034 regardless of whether today's climate data is used or that available only up to 1974, raising the question of whether the Keeling curve can be flattened in ten years.

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I. BACKGROUND

As recently pointed out by Betts *et al*, "Assessing global mean temperature rise using the average warming over the previous one or two decades will delay formal recognition of when Earth breaches the Paris agreement's 1.5 °C guard rail."[1]

Having worked extensively in formal methods[2], the word "formal" caught my eye. It occurred to me that a mixed formal/empirical notion of climate might help here, loosely analogous to Newton's formal/empirical notion of gravitational force.

We can regard Newton as having postulated formally that the gravitational force between two point masses distance r apart was a function of their mutual gravitational potentials $-m_1/r$ and $-m_2/r$, namely their product up to an unknown constant G. 111 years later, Henry Cavendish gave the first empirical determination of G, improved to six significant digits during the following two centuries, always at laboratory scales. G is assumed constant at far larger scales than those determinations, but considerations of dark matter have raised questions about galaxies more remote than those of our local cluster.[3].

II. FORMAL CLIMATE

Our counterpart here is a formal definition of planetary *industrial greenhouse blocking*, igb, as an analytic function of time t, namely $\mathsf{igb}(t) = \log_2(1 + 2^t)$ up to an unknown constant. Here t is the time since industrial atmospheric CO2 (iCO2) doubled the planet's preindustrial level of atmospheric CO2, in units of the time taken to double iCO2, while $\log_2$ reflects how CO2's absorption lines block about 5 cm^{-1} (150 GHz) of the atmospheric window per octave of line strengths[4]. Whereas Newton had several planets to confirm his formal equation for gravity, until we discover a newly industrialized planet with t in the vicinity of zero, we will only have ours to go on.

As pointed out in 2009 by Hofmann *et al*[5], iCO2 has been growing exponentially, doubling every 35 years or so. Based on ice core data along with more recent Mauna Loa data, we were surprised to find that scaling iCO2 by e every 50.00 years fit even more accurately than doubling in $35 = 105/3$ years, corresponding to doubling in almost exactly $104/3$ years or compounding annually at 2.02%. Today t would appear to be -1 in these doubling units. That is, unless something unexpected happens to climate in the next few decades, iCO2 will double from its 140 ppm in 2023 to 280 ppm in 2058. Writing $t = 3(y - 2058)/104$, an equivalent formula is $\mathsf{igb}(y) = \log_2(1 + 1.0202^{y-2058})$ where y is the Christian Era year, 2024 as of this writing, using $2^{3/104} = 1.0202$. This can also be written as $\log_2((280 + 1.0202^{y-1776})/280)$ where 280 is preindustrial CO2 in units of parts per million and $1776 = 2058 - 104\log_2(280)/3$. (By coincidence, 1776 happens to be the year Scottish inventor James Watt sold his first coal-fired steam engine to Scotland's Carron Company.)

III. CENTENNIAL CLIMATE

There are at least nine named subdecadal, decadal, and multidecadal ocean oscillations tending to mask subcentennial climate trends[6]. There are also multimillennia climate contributors such as the Milankovitch cycles, and even longer term effects such as the polar glaciation and subsequent deglaciation following the Paleo-Eocene Thermal Maximum.

Our thesis is that when tuning the dial of the climate spectrum from multidecadal to multimillenial, we pass through a sweet spot where the only climate signals are a minor 130-year solar oscillation dwarfed by an exponential rise in industrial CO2. On the high frequency side of this sweet spot is the so-called 65-year multidecadal Atlantic Multidecadal oscillation, and on the low side, the Nyquist limit resulting from a lack of sufficient global

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mean surface temperature data prior to 1850.

We propose to use this sweet spot to define centennial climate. This of course is just one possible definition; we would be interested to see arguments for a substantially different formally defined year.

We isolate the sweet spot with a trapezoidal filter, equivalent to a 65-year box filter composed with an 11-year one, designed to block all known ocean oscillations as well as the 11-year solar cycle. We designate as centennial data cD all data D to which this filter is applied, from 1850 to the present, for definitions of "the present" from 1974 to 2024 in increments of ten years. We apply it to HadCRUT5 yielding cHC; $GB = \log_2(CO2)$ data from Antarctic ice cores to 1958 and Mauna Loa data thereafter yielding cGB for centennial greenhouse blocking; and TSI data as reconstructed from sunspot numbers by G. Kopp and L. Svalgaard yielding cTSI.

IV. TRANSIENT GREENHOUSE AND SOLAR RESPONSES

As our counterpart of Cavendish's determination of G using swinging lead balls, we use multiple linear regression to fit cGB and cTSI to cHC, yielding response coefficients R_g and R_s for Transient Greenhouse Response and Transient Solar Response respectively that minimize the variance, and hence standard deviation σ, of $cHC - (cGW + cSW)$. Here $cGW = R_g \times cGB$ is centennial Greenhouse Warming associated with outgoing longwave radiation with wavelengths λ on the order of 10 μm, while $cSW = R_s \times cTSI$ is centennial Solar Warming with λ twenty times shorter. Taking "the present" to be 2024, we obtain $R_g = 2.232$, $R_s = 0.244$, and $\sigma = 3.0$ mK. This strikingly low σ can be better appreciated from the plot of $cHC - cGW$ vs. cSW in Figure 1.

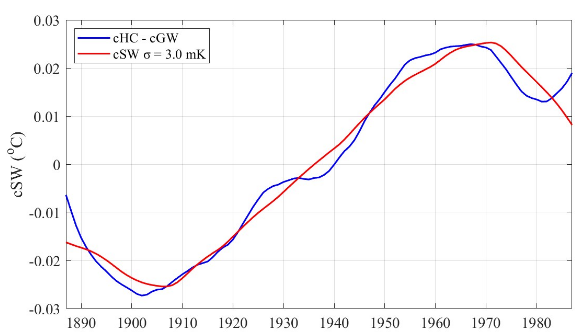

FIG. 1. cHC - cGW vs. cSW

We define $fcc(y)$, formal centennial climate as a function of Gregorian year y, as $fcc(y) = R_g \times \mathrm{igb}(y) = R_g \log_2(1 + 1.0202^{y-1776})$.

The coefficient R_s for solar warming is notably absent from our definition, raising the question of why it is needed at all.

The role of R_s is to prevent attributing any warming by the Sun to CO2. Had we used simple linear regression to fit cCO2 to cHC, what we would have obtained would have been TCR, transient climate response, naively attributing solar warming to greenhouse warming, thereby increasing the transient response to the greenhouse effect by some 10%, and moreover with a worse fit of about $\sigma = 10$ mK. Since R_s has presumably been around for millions of years, and its contribution to the difference between TCR and R_g would appear to be merely an accident of the phase of that particular solar oscillation, it seemed unnecessary to incorporate it into our definition of $fcc(y)$. We suggest viewing our notion of R_g as simply TCR after suitably taking TSI into account.

V. PREDICTABILITY

As our counterpart of the two centuries of refinements of Cavendish's original determination of G, we perform all of the above for six values of "the present", from 1974 to 2024, ten years at a time, ignoring all data from "the future". Figure 2 compares HadCRUT5 (blue) vs. AGW (red) in four ways. (i) and (iii): centennially filtered. (iii) and (iv): formal climate in place of observed CO2 and TSI. Figure 2(a) uses only data up to 1974, 2(b) up to 2024, i.e. all data to date.

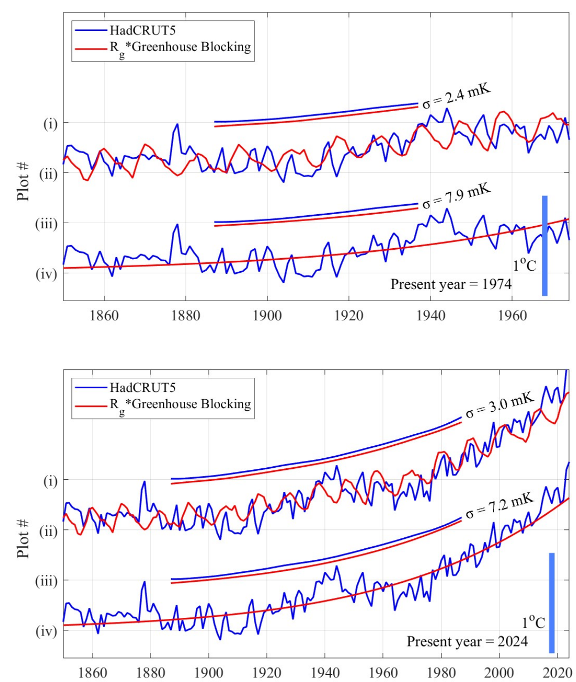

FIG. 2. HC vs. GW (a) in 1974 (b) in 2024

Table I gives R_g, R_s, σ, and σ' for (i) and (iii) of Figure 2 respectively, along with $fcc(y)$ and $fcc(y+1)$ for $y =$ 1970, 2024, 2033, 2093.

TABLE I. Six decades of fcc predictions

PRSNT	R_g	R_s	σ	σ'	1970	1971	2024	2025	2033	2034	2093	2094
1974	2.182	0.288	2.4	7.9	0.500	0.509	1.290	1.312	1.492	1.516	3.473	3.515
1984	2.179	0.267	2.4	7.7	0.499	0.509	1.289	1.310	1.491	1.515	3.469	3.511
1994	2.184	0.280	2.5	8.1	0.500	0.510	1.292	1.313	1.494	1.518	3.476	3.519
2004	2.190	0.280	2.3	7.9	0.502	0.511	1.295	1.316	1.498	1.522	3.485	3.527
2014	2.162	0.296	2.3	8.8	0.495	0.504	1.279	1.300	1.479	1.503	3.441	3.483
2024	2.232	0.244	3.0	7.2	0.511	0.521	1.320	1.341	1.526	1.551	3.552	3.595

Note that R_g remains close to 2.2 °C per doubling of CO2 modeled formally as $280 * (1 + 1.0202^{y-2058})$ ppm as half a century of additional data since 1974 comes to hand. The MATLAB scripts producing these values, obtainable from the author's website[7], are based solely on global annual HadCRUT5, CO2 from Antarctic ice cores before 1958 and Mauna Loa measurements thereafter, and total solar irradiance TSI as reconstructed independently by G. Kopp and L. Svalgaard.

Whereas Newton's G was refined during the two centuries after Cavendish, R_g was not so much improved as shown to be relatively stable as time produced further data. If iCO2 were at some future time to grow significantly more slowly than its present CAGR of about 2%, this notion of formal climate would need to be revisited. To date there has been no sign of any such deceleration of rising CO2.

ACKNOWLEDGMENTS

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